# *L*-Band Ferromagnetic Resonance Experiments at High Peak Power Levels*

E. SCHLÖMANN†, J. H. SAUNDERS†, AND M. H. SIRVETZ†

*Summary*—Ferromagnetic resonance absorption at high peak power levels has been observed at 1300 mc in yttrium-gadolinium garnets and in a nickel ferrite-aluminate. In agreement with theoretical predictions, the critical field characterizing the onset of nonlinear effects, in a series of yttrium-gadolinium garnet disks of a given shape, was found to be very sensitively dependent on the gadolinium content. Similarly, for samples of a given composition, the critical field strength was sensitively dependent on the shape of the sample in agreement with theoretical predictions. At moderate power levels the susceptibility varies linearly with the square of the RF magnetic field strength over an appreciable range. This result can be understood in terms of an extension of Suhl's theory. The results can be used to predict the high power performance of these materials when used in isolators.

## I. Introduction and Theory

In many important applications of ferrite devices high peak and average power levels are encountered. With Curie temperatures of the order of 200°C, and suitable provision for cooling, it would seem that isolators can be developed to handle any average power of interest (up to tens of kilowatts). With respect to peak power levels, on the other hand, the situation is not so simple, and it is known that the response of ferrite materials becomes nonlinear above a certain critical level[1,2] which for a given frequency depends on the linewidth, the saturation magnetization, and the shape of the specimen.

These effects manifest themselves as a reduction of the absorption at resonance and, under certain conditions, as an increase of the absorption away from resonance. The nonlinear effects usually set in at a fairly well defined threshold, and their presence very often interferes with the performance of ferrite devices such as isolators, phase shifters, and circulators. It is, therefore, important to investigate the possibilities of reducing the nonlinear effects.

The general features of the observed phenomena can be understood quite well in terms of a theory developed by Suhl.[3,4] According to this theory the nonlinear effects are due to the fact that certain spin waves become excited as soon as the amplitude of the uniform mode (which is driven by the applied microwave field) exceeds a certain critical value. Two possible mechanisms can be distinguished. For one of them, the unstable spin waves have half the frequency of the applied signal; for the other, they have the same frequency as the applied signal. We shall refer to these two mechanisms as nonlinear of first and second order. It has been shown by Suhl[3,4] that the first order effect has an inherently lower threshold than the second order effect (*i.e.*, it sets in at a lower value of the amplitude of the uniform mode, hence a lower power level). It has also been shown by Suhl,[3,4] however, that the first order effect can be suppressed if the experimental conditions are such that all spin wave frequencies are higher than half the signal frequency. For spheroidal samples this condition is realized at resonance if the signal frequency exceeds a "characteristic frequency"

$$\omega_c = 2\gamma 4\pi M_s N_\perp. \tag{1}$$

Here $\gamma$ is the gyromagnetic ratio, $M_s$ the saturation magnetization and $N_\perp$ the transverse demagnetizing factor of the sample (equal to $\frac{1}{3}$ for a sphere). At frequencies higher than this value only the second order effect can produce a change in the susceptibility at resonance and the first order effect gives rise to a "subsidiary" absorption peak at field strengths below those required for resonance. If the sample is biased for resonance, the critical RF field strength at which the nonlinear effects set in is, according to Suhl,

$$\begin{aligned} h_{\text{crit}} &= \Delta H \cdot \frac{\Delta H_k}{4\pi M} F && \text{if } \omega < \omega_c \\ &= \Delta H \cdot \sqrt{\frac{\Delta H_k}{4\pi M_s}} && \text{if } \omega > \omega_c. \end{aligned} \tag{2}$$

Here $\Delta H$ is the observed linewidth (total width of the resonance curve at half its maximum height) and $\Delta H_k$ a linewidth characteristic of the unstable spin waves. $\Delta H_k$ is believed to represent the intrinsic losses of the material, whereas very often $\Delta H$ is determined to a large extent by line broadening processes which arise from inhomogeneities such as the randomness in the distribution of the various magnetic ions over the available sites in the spinel lattice,[5] polycrystallinity,[6,7] and surface roughness.[8] The constant $F$ in (2) is of the order

* Manuscript received by the PGMTT, August 3, 1959.
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[1] R. W. Damon, "Relaxation effects in the ferromagnetic resonance," *Rev. Mod. Phys.*, vol. 25, pp. 239–245; January, 1953.

[2] N. Bloembergen and S. Wang, "Relaxation effects in para- and ferromagnetic resonance," *Phys. Rev.*, vol. 93, pp. 72–83; January, 1954.

[3] H. Suhl, "The nonlinear behavior of ferrites at high microwave signal levels," Proc. IRE, vol. 44, pp. 1270–1284; October, 1956.

[4] H. Suhl, "The theory of ferromagnetic resonance at high signal powers," *J. Phys. Chem. Solids*, vol. 1, no. 1, p. 209; 1957.

[5] A. M. Clogston, H. Suhl, L. R. Walker, and P. W. Anderson, "Ferromagnetic resonance line width in insulating materials," *J. Phys. Chem. Solids*, vol. 1, no. 3, pp. 129–136; 1956.

[6] S. Geschwind and A. M. Clogston, "Narrowing effect of dipole forces on inhomogeneously broadened lines," *Phys. Rev.*, vol. 108, pp. 49–53; October, 1957.

[7] E. Schlömann, "Spin-wave analysis of ferromagnetic resonance in polycrystalline ferrites," *J. Phys. Chem. Solids*, vol. 6, no. 2/3, pp. 242–256; 1958.

[8] R. C. Le Craw, E. G. Spencer, and C. S. Porter, "Ferromagnetic resonance line width in yttrium iron garnet single crystals," *Phys. Rev.*, vol. 110, pp. 1311–1313; June 15, 1958.

of unity if $\omega$ is sufficiently smaller than $\omega_c$ and becomes very large as $\omega$ approaches $\omega_c$. In (2) the RF field is assumed linearly polarized. If the experiment is performed on small samples of ellipsoidal shape, the linewidth should be inferred from the effective susceptibility (relating the RF magnetization to the RF magnetic field outside the sample) and $h_{\mathrm{crit}}$ represents the cavity field in the absence of the sample.

It is obvious from the preceding remarks that at high microwave frequencies (such as $X$-band) and for conventional ferrimagnetic materials, the saturation of the resonance usually requires the second order process. At low microwave frequencies (such as $L$-band), however, the first order process is usually predominant unless special precautions are taken to suppress it. Eq. (1) shows that, in order to achieve this aim, one must reduce the saturation magnetization and/or use thin samples magnetized perpendicular to their planes. If the signal frequency is approximately equal to the characteristic frequency given by (1), the critical field strength should be very sensitive to small changes in the signal frequency, the magnetization, and the sample shape.

Suhl has also calculated the stationary behavior at power levels beyond the critical field given by (2). His original calculations with respect to this problem apply only to the first order process. He has shown that under these conditions the susceptibility at resonance should vary inversely with the amplitude of the RF magnetic field, and this has been confirmed experimentally.[3,4]

One of the present authors[9] has recently extended Suhl's work to a calculation of the stationary response at high power levels for the case in which the second order process accounts for the saturation of the resonance. The principal innovation of the new theory consists in a more detailed consideration of line broadening mechanisms. In Suhl's theory the effect of dissipation is taken into account by means of phenomenological damping terms in the equations of motion. This damping is very important because it stabilizes the spin waves in the presence of the time-varying coupling produced by the excitation of the uniform mode. It can be shown that this phenomenological description is not sufficient if the linewidth is predominantly caused by inhomogeneity broadening. This situation is encountered in many of the ferrites where the availability of various sites in the spinel lattice gives rise to an inhomogeneously broadened line as discussed by Clogston *et al.*[5] It is also encountered in polycrystalline ferrites where crystalline anisotropy, in conjunction with the polycrystalline character of the sample, gives rise to similar effects.[6,7] In these cases, the dissipation must be taken into account in a more detailed fashion, because the results so obtained differ appreciably from those obtained with a phenomenological damping term.

This can be understood in a fairly simple way. In the absence of inhomogeneity interaction the motion of the spin-wave amplitudes is determined by the time-varying coupling produced by the large excitation of the uniform mode. The spin waves are not excited at low powers and become unstable at a well defined threshold.

Inhomogeneities give rise to a linear coupling between the uniform mode and the spin waves. Because of this coupling, those spin waves that have the right frequency are slightly excited by an applied microwave field even at very low powers. At higher powers the same spin waves are simultaneously subject to a time-varying coupling produced by the large amplitude of the uniform mode. The situation is then similar to that encountered in parametric amplifiers. The excitation of the important spin waves caused by the inhomogeneity interaction is greatly enhanced by the time-varying coupling. Even at powers below those required for instability some of the spin waves are excited to a comparatively high level. The excitation varies smoothly as the power is increased beyond the point at which instability would occur in the absence of inhomogeneity interaction.

The results of the revised theory can be stated as follows: at moderate power levels the susceptibility at resonance should vary linearly with the square of the amplitude of the RF magnetic field

$$\frac{\chi''}{\chi_0''} = 1 - C\left(\frac{h}{\Delta H}\right)^2. \tag{3}$$

Here $\chi_0''$ is the susceptibility at low power levels. The constant $C$ depends on the location of the uniform mode in the spin-wave band, the strength of the inhomogeneity interaction, and the correlation length characteristic of the dominant scattering process. A numerical calculation of this constant has not been possible because of mathematical difficulties and because the physical information necessary for such a calculation is not available at present. It is interesting to note that $C$ is not necessarily positive. The experimentally observed values lie in the range between $+150$ and $-30$. At higher power levels, the susceptibility should vary inversely with the amplitude of the RF magnetic field

$$\frac{\chi''}{\chi_0''} = \frac{h_\infty}{h} \tag{4}$$

where

$$h_\infty = \Delta H \sqrt{\frac{\Delta H_k}{4\pi M_s}}\,. \tag{5}$$

In (3)—(5), $\chi''$ is the effective susceptibility and $h$ is the cavity field (assumed linearly polarized) in the absence of the small sample. The derivation of (4) is based on certain approximations which cease to be applicable if the power level is very high. It may, therefore, be expected that this equation breaks down at very high power levels. The experimental results that have so far been obtained, usually agree quite well with (4), even at the highest available power levels.

[9] E. Schlömann, "Ferromagnetic resonance at high signal powers," *Bull. Amer. Phys. Soc.*, vol. 4, p. 53; January, 1959.

Fig. 1 demonstrates the theoretical dependence of the resonance susceptibility on the power level. The broken line represents the results obtained without inhomogeneity broadening. As Suhl has shown, the susceptibility should be constant up to $h=h_{crit}$ (which equals $h_\infty$), in this case. The curve shown for $h>h_\infty$ represents the $1/h$ dependence that one may expect in analogy to Suhl's results concerning the first order effect. The full line shows in a qualitative way the results obtained with the inclusion of inhomogeneity broadening. It is seen that the sharp break which previously characterized the onset of the nonlinear effects has been replaced by a gradual decline. The curve shown is valid for the special case in which $C(h_\infty/\Delta H)^2=\frac{1}{4}$. For different values of this parameter the curves will, of course, be different.

Suhl[10] has recently investigated the saturation of the resonance through the second order process taking into account inhomogeneity broadening in the manner suggested by Schlömann.[9] His results can be represented by a curve very similar to the one shown in Fig. 1. His curve also decreases as $1/h$ at very high powers. But the behavior at moderate powers is of the form $1-ch^4$.

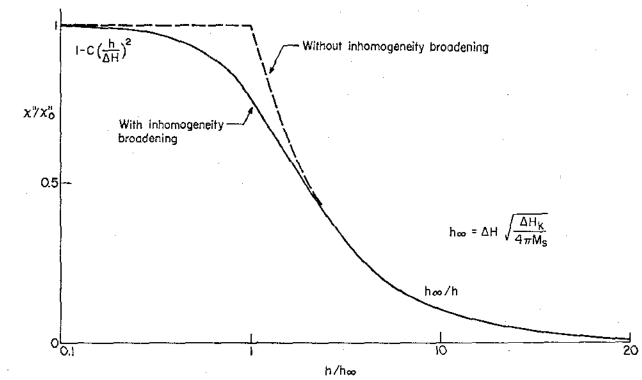


Fig. 1—Theoretical dependence of the susceptibility at resonance on the power level.

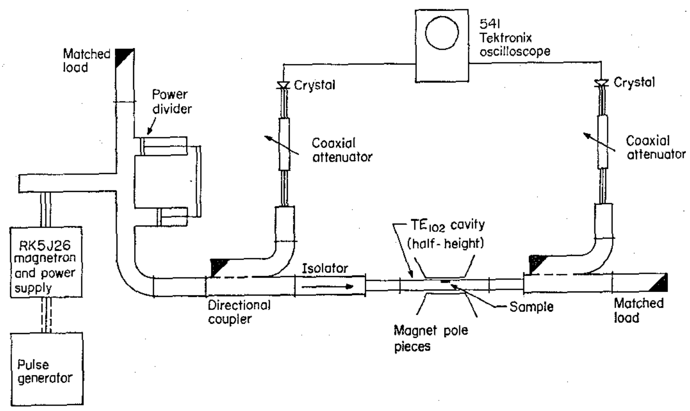


Fig. 2—Experimental equipment for high power measurements at $L$-band.

## II. Experimental Procedure

Fig. 2 shows a schematic diagram of the apparatus used for observing at 1300 mc the ferromagnetic resonance absorption under conditions of high peak power levels. The measurement is made on small specimens in a rectangular $TE_{102}$ transmission cavity of high unloaded $Q$. By choosing an appropriate set of inductive irises, the loaded $Q$ is adjusted to a value between 300 and 2000 according to the strength of the absorption being measured and the range of RF field values required. A pulsed magnetron provides $\frac{1}{2}$ megawatt pulses to a power divider which is adjusted to give constant amplitude cavity output as the dc magnetic field is varied. The incident power is measured at several points on the resonance curve by means of a directional coupler, a coaxial cutoff attenuator, and a crystal detector. The setting of the attenuator for constant scope deflection removes the dependence on the crystal detection law. A duty cycle of the order of $6\cdot 10^{-5}$ is used to eliminate spurious effects due to heating of the specimen. The pulse length was 4 $\mu$sec.

## III. Results

It was explained in Section I that a significant change in the critical field should be expected if the experimental conditions are changed in such a way that the first order nonlinear process is allowed in one case and forbidden in the other. Spencer *et al.*[11] and Le Craw *et al.*[12] have observed this effect by changing the frequency and the temperature, respectively. In the present experiments the same effect has been observed by changing the composition (thus changing the saturation magnetization) and by changing the shape of the specimen. In Fig. 3 the susceptibility at resonance is plotted as a function of $1/h$ for a series of yttrium-gadolinium garnet disks of equal shape. In this representation the approximately linear dependence of $\chi''$ on $1/h$ is most apparent, and the critical field $h_\infty$ can be obtained by a simple extrapolation. It is seen that the critical field increases rapidly with increasing gadolinium content and is approximately twenty times higher for the sample with the largest gadolinium content than it is for pure yttrium garnet. The linewidth also increases in this series but only by a factor of less than three.

In these experiments a clear $1/h$ dependence of the susceptibility at very high power levels has not been observed. This is probably due to extraneous effects. The samples used for these experiments are disks and not ellipsoids as assumed in theory. In addition the internal dc magnetic field at resonance is rather small, so that the sample may not be completely magnetized. It is believed that these two facts account in large measure for the apparent residual susceptibility at very high power levels. In similar experiments at $X$-band[13] it has usually been found that the susceptibility varies as $1/h$ at high power levels.

[10] H. Suhl, to be published in *J. Appl. Phys.*

[11] E. G. Spencer, R. C. Le Craw, and C. S. Porter, "Ferromagnetic resonance in yttrium iron garnet at low frequencies," *J. Appl. Phys.*, vol. 29, pp. 429–430; March, 1958.

[12] R. C. Le Craw, E. G. Spencer, and C. S. Porter, "Ferromagnetic resonance and nonlinear effects in yttrium iron garnet," *J. Appl. Phys.*, vol. 29, pp. 326–327; March, 1958.

[13] J. J. Green and E. Schlömann, "High power ferromagnetic resonance at X-band in polycrystalline garnets and ferrites," this issue, pp. 100–103.

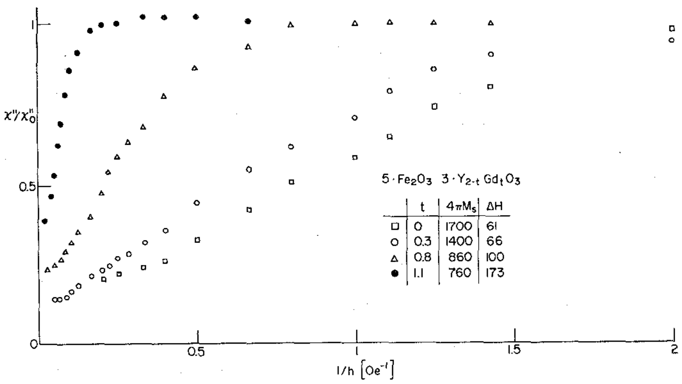


Fig. 3—Resonance absorption at high power levels for various yttrium-gadolinium garnets. Measurements at 1300 mc on disks ($\frac{25}{32}$-inch diameter by 0.050-inch thick).

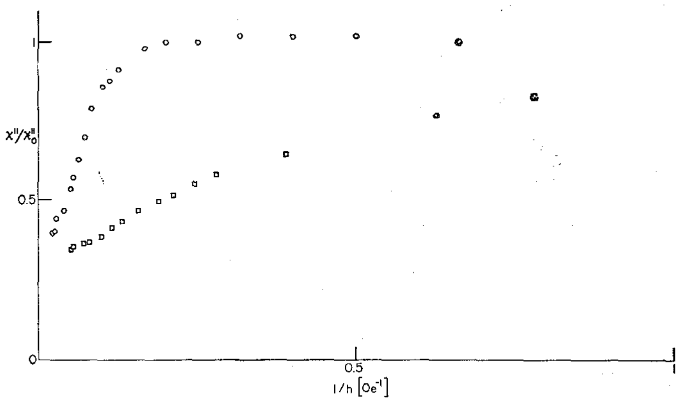


Fig. 4—Resonance absorption at high power levels for a disk (○) $\frac{25}{32}$-inch by 0.050-inch thick and a sphere (□) $\frac{5}{16}$-inch-diameter of a yttrium-gadolinium garnet (5 $Fe_2O_3 \cdot 3Y_{0.9}Gd_{1.1}O_3$). Measurements taken at 1300 mc, $4\pi M = 760$, $\Delta H = 173$ Oe.

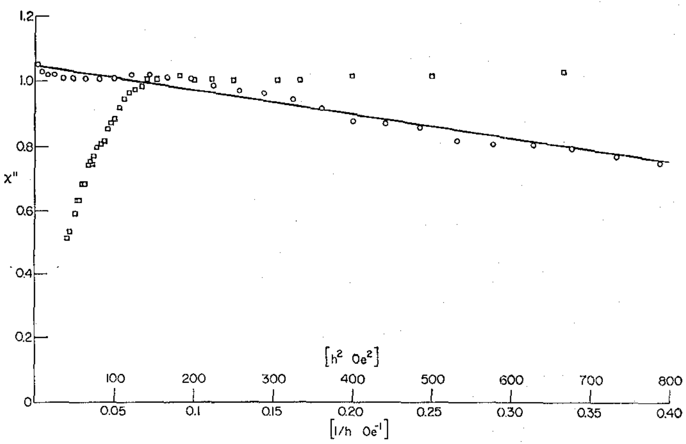


Fig. 5—Resonance absorption as a function of power level for a nickel ferrite-aluminate developed in this laboratory ($4\pi M_s$ 475 gauss, $\gamma = 2.1$ Mc/Oe, $\Delta H = 290$ Oe at $X$-band). Measurements taken at 1300 mc on a disk ($\frac{5}{8}$-inch diameter by $\frac{3}{16}$-inch thick). ○ refers to $h^2$ scale, □ to $1/h$ scale.

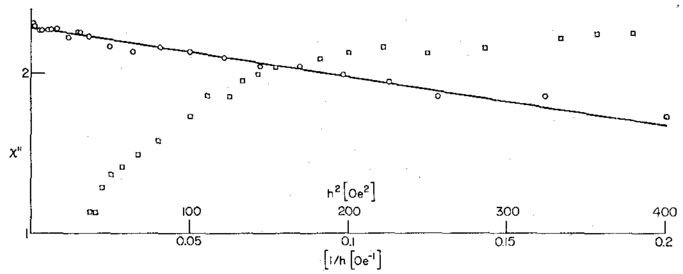


Fig. 6—Resonance absorption as a function of power level for a yttrium-gadolinium garnet (5 $Fe_2O_3 \cdot 3Y_{0.9}Gd_{1.1}O_3$: $4\pi M_s = 710$ gauss, $\Delta H = 200$ Oe). Measurements taken at 1300 mc on a disk ($\frac{25}{32}$-inch diameter by 0.050-inch thick). ○ refers to $h^2$ scale, □ refers to 1/h scale.

When the complete resonance line ($\chi''$ vs dc magnetic field) was measured it was observed that with increasing power level the absorption maximum shifted toward lower fields. In the case of the yttrium garnet, the resonance line became very steep on the low field side at an RF field amplitude of approximately 6 Oe. This "fold-over effect" has been predicted theoretically by Anderson and Suhl[14] and has previously been observed by M. Weiss.[15] In all other samples except the yttrium garnet the effect was not noticeable. A theoretical analysis showed that for the values of saturation magnetization and linewidth ($\Delta H$ and $\Delta H_k$) encountered in these experiments, the fold-over effect should not play an important role.

Fig. 4 shows the susceptibility as a function of $1/h$ for two samples of the same yttrium-gadolinium garnet: one, a thin disk; the other, a sphere. Again the large difference in the critical field strength associated with the transition from the first (sphere) to the second order nonlinear process (disk) is apparent.

If the susceptibility is plotted vs $h^2$ on a linear scale the linear dependence of the susceptibility on $h^2$ at moderate power levels becomes apparent. Figs. 5 and 6 show two examples. In both cases the second order nonlinear process accounts for the saturation of the resonance. It is seen that the linear law agrees quite well with the data over an appreciable range of power levels. In the present cases the initial third of the decline of the resonance susceptibility is governed by the linear law. The constants $C$ of (3) for the two cases are 29 (for the nickel ferrite-aluminate) and 11 (for the yttrium-gadolinium garnet). A different yttrium-gadolinium garnet of the same nominal composition showed an initial slight increase of the susceptibility with power level (*i.e.*, $C < 0$).

## IV. Conclusion

It may be seen from the last two figures that $h_\infty$ for these materials is of the order of 25 Oe. This corresponds to approximately 1.7 megawatts in $L$-band waveguide of half the conventional height.[16] It should be remembered that a wave traveling in the forward direction of an isolator does not excite the resonance. Only the reflected power must be absorbed by the isolator.

[14] P. W. Anderson and H. Suhl, "Instability in the motion of ferromagnets at high microwave power levels," *Phys. Rev.*, vol. 100, pp. 1788–1789; December 15, 1955.

[15] M. T. Weiss, "Microwave and low-frequency oscillation due to resonance instabilities in ferrites," *Phys. Rev. Letters*, vol. 1, pp. 239–241; October, 1958.

[16] Half-height waveguide is generally used for isolators at this frequency.

With mismatches of the sort encountered in practical applications (typically less than two to one), an isolator using the materials discussed in this paper should function satisfactorily at peak power levels of several megawatts, if the necessary precautions are taken to suppress the first order nonlinear effect. Experiments using typical isolator configurations instead of the resonance cavity have shown that the onset of nonlinear effects in isolators can be predicted with fair accuracy from cavity measurement.

A further improvement of the power handling capacity of microwave ferrites appears quite feasible. In particular, it is not difficult at all to increase the linewidth and thus increase the critical field ($h_{crit}$ or $h_\infty$). This is not a promising line of attack, however, since the ratio of reverse to forward attenuation of resonance isolators decreases rapidly with increasing linewidth.[17] To improve the power-handling capacity one must, therefore, according to (5), decrease the saturation magnetization and/or increase the spin-wave linewidth $\Delta H_k$. This can generally be achieved by suitable substitution of the magnetic ions in ferrites of the spinel and garnet type.

ACKNOWLEDGMENT

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[17] B. Lax, "Frequency and loss characteristics of microwave ferrite devices," Proc. IRE, vol. 44, pp. 1368–1386; October, 1956.

# High Power Ferromagnetic Resonance at X-Band in Polycrystalline Garnets and Ferrites*

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*Summary*—**Resonance experiments have been performed at X-band on spherical samples of polycrystalline yttrium garnet, yttrium-gadolinium garnet, yttrium-holmium garnet and nickel-cobalt ferrite. The RF field strength extended up to 60 Oersted. In the case of yttrium garnet the samples differed considerably in density and hence in linewidth. At fairly low power levels the susceptibility at resonance varies linearly with the square of the RF magnetic field strength. At high power levels the susceptibility is inversely proportional to the amplitude of the microwave magnetic field. The "spin-wave linewidth" $\Delta H_k$ is inferred by extrapolation from the behavior at very high powers. It is found that $\Delta H_k$ is, to a large extent, independent of the linewidth $\Delta H$ observed by the usual low power experiments. In particular $\Delta H_k$ was found to be essentially the same (approximately 4 Oe) for all yttrium iron garnets (single crystals and polycrystals with linewidth varying between 1.8 Oe and 450 Oe). On the other hand, $\Delta H_k$ increases very rapidly if the yttrium is partially substituted by holmium ($\Delta H_k \sim 11$ Oe for 1 per cent substitution.)**

## I. Introduction

It was explained in the preceding paper[1] that at X-band frequencies and for conventional ferrimagnetic materials, the saturation of the resonance line involves the excitation of spin waves which have the same frequency as the signal (second-order nonlinear process). In addition, at these frequencies, one observes a subsidiary absorption peak below the main resonance which is attributed to the excitation of spin waves with frequencies equal to half the signal frequency (first-order nonlinear process). For spherical samples of the materials investigated in this paper the subsidiary peak lies considerably below the main resonance and (for this reason) becomes noticeable only at quite high power levels. This means that one never encounters a situation in which both nonlinear processes (first and second order are simultaneously important. Experiments at X-band are, therefore, better suited for a detailed investigation of the second-order nonlinear process than the L-band experiments reported in the preceding paper.[1]

The materials used in this investigation belong to four different families. One of these families comprises yttrium iron garnets of varying densities and hence varying linewidths (between 47 and 450 Oe). Two other families are derived from yttrium iron garnet by partial substitution of gadolinium or holmium for the yttrium. The fourth family is derived from nickel ferrite by partial substitution of cobalt for nickel.

## II. Experimental Procedure

Ferromagnetic resonance measurements were made at 9250 mc using a $TE_{202}$ transmission cavity. The samples were mounted on a quartz rod and placed in the cavity at a point of maximum RF magnetic field. Sample absorption as a function of dc magnetic field was determined from the change in incident power necessary to maintain a constant transmitted power. The magnetic field strength at the sample was obtained from

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[1] E. Schlömann, J. Saunders, and M. Sirvetz, "L-band ferromagnetic resonance experiments at high peak power levels," this issue, pp. 96–100.